

"MINUS C" SYMMETRY IN CLASSICAL AND QUANTUM THEORIES

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Abstract

It is shown that the transformations of the charge conjugation in classical electrodynamics and in quantum theory can be interpreted as the consequences of the symmetry of Maxwell and Dirac equations with respect to the inversion of the speed of light $t \rightarrow t, \mathbf{x} \rightarrow \mathbf{x}, c \rightarrow -c$. The elements of physical interpretation are given.

1 Introduction

A symmetry approach plays a large part in theoretical physics. Below we call attention to the discrete symmetries. Discrete transformations are well known in particle physics [1], in quantum field theory [2], in nuclear physics [3], in quantum physics by studying systems with periodic changing parameters [4]. The space inversion $P(\mathbf{x} \rightarrow -\mathbf{x})$ reflecting the right - left symmetry of the 3-dimensional space of events, the time reversal $T(t \rightarrow -t)$ describing the symmetry connected with change of sign of time, the charge conjugation C connected with symmetry of particle - antiparticle, the wave function transformations of the type $\Psi_\epsilon(t + T) = e^{-i\epsilon T/\hbar} \Psi_\epsilon(t)$ are the examples of the such symmetries [1-4]. Recently has been established also what beyond that point an additional discrete transformation exists. It is the inversion of the speed of light $t \rightarrow t, \mathbf{x} \rightarrow \mathbf{x}, c \rightarrow -c$ or $x^0 \rightarrow x^0, \mathbf{x} \rightarrow \mathbf{x}, c \rightarrow -c$ where $x^0 = ct$ [5], [6]. Let us designate the appropriate symmetry by the symbol Q and name it the "minus c" or " $c \rightarrow -c$ " symmetry. The equation of light cone $c^2 t^2 - \mathbf{x}^2 = 0 \rightarrow (-c)^2 t^2 - \mathbf{x}^2 = 0$ may demonstrate the example of existing of such symmetry. "Minus c" symmetry is also inherent in the D'Alembert equation, the Maxwell equations, the equation of movement of a charge particle in electromagnetic field, the Schrödinger and Klein-Gordon-Fock equations [5]. It is shown also that " $c \rightarrow -c$ " symmetry is connected closely with the charge conjugation C , which may be interpreted as the evidence of invariance of Dirac equation with respect to the QPT composition [6]. The purpose of the present work is the further study of relation between the transformation of charge conjugation C and " $c \rightarrow -c$ " inversion and possible interpretation of obtained results.

2 Group of discrete transformations of space, time, the speed of light G_8

Let us introduce the 5-dimensional space of events $V^5(x^0, \mathbf{x}, c)$ and two hyperplanes with $c = +3.10^{10} \text{ cm/s}$ and $c = -3.10^{10} \text{ cm/s}$ in it and construct four matrices of dimension 5×5 :

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 1 \end{pmatrix}; \alpha^T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 1 \end{pmatrix}; \alpha^P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & 1 \end{pmatrix}; \alpha^Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (1)$$

Here I is the 3X3 unit matrix; $\alpha^{T^2} = \alpha^{P^2} = \alpha^{Q^2} = E$; $[\alpha^T, \alpha^P] = [\alpha^T, \alpha^Q] = [\alpha^P, \alpha^Q] = 0$. Matrices and their products form the cyclic Abelian group and induce the 8-dimensional group of discrete transformations of time, space and speed of light G_8 in the 5-space of events $V^5(x^0, \mathbf{x}, c)$:

$$x^{a'} = (\alpha^{T^k} \alpha^{P^l} \alpha^{Q^m})^{ab} x^b; \quad a, b = 0, 1, 2, 3, 5 \quad (2)$$

The matrices $\alpha^P, \alpha^T, \alpha^Q$ are in conformity with operators P, T, Q acting on field functions of the equations studied at replacement of variables (2).

We consider the place of group G_8 and its subgroups in symmetry theory of classical and quantum equations.

3 Discrete symmetry of Maxwell equations

We take the one-charge Maxwell equations and shall consider them in the 5-dimensional space $V^5(x^0, \mathbf{x}, c)$ on hyperplanes with $+c$ and $-c$. The each hyperplane can be interpreted as the 4-Minkowski subspace as far as the sign of speed of light does not influence on the metric tensor $g_{ab} = \text{diag}(+, -, -, -)$ in view of raising to second power the differentials of coordinates in the expression of squared interval $ds^2 = dx^{0^2} - d\mathbf{x}^2 = (-dx^0)^2 - d\mathbf{x}^2$. We have on $+c$ hyperplane as follows:

$$\begin{aligned} \nabla X \mathbf{H} - \partial_0 \mathbf{E} &= 4\pi \mathbf{J}; \quad \nabla \cdot \mathbf{H} = 0 \\ \nabla X \mathbf{E} + \partial_0 \mathbf{H} &= 0; \quad \nabla \cdot \mathbf{E} = 4\pi \rho \end{aligned} \quad (3)$$

Here $x^0 = ct$, $x^{1,2,3} = x, y, z$, \mathbf{E}, \mathbf{H} are the electrical and magnetic field; ρ is the density of electric charge e ; $\mathbf{J} = \rho \mathbf{v}/c$ is the density of current; \mathbf{v} is the speed of charge; $\mathbf{E} = -\partial_0 \mathbf{A} - \nabla \phi$, $\mathbf{H} = \nabla X \mathbf{A}$; $A = (\phi, \mathbf{A})$ is the 4-dimensional potential.

The statement takes place: the group of transformations (2) is the group of discrete symmetry of Maxwell equations.

The proof is convenient to conduct with help of 16-dimensional function $\Phi^e(x^0, \mathbf{x}, c) = \text{column}(0, \mathbf{E}, 0, \mathbf{H}, \rho, \mathbf{j}, \phi, \mathbf{A})$ wrote on the $+c$ hyperplane and labeled by the discrete top index e (electric charge). The Maxwell equations are transformed to themselves if the function Φ^e is transformed by the rules:

$$\begin{aligned} T_1 \Phi^e(x^0, \mathbf{x}, c) &= \Phi_{T_1}^e(-x^0, \mathbf{x}, c) = \text{column}(0, +\mathbf{E}, 0, -\mathbf{H}, +\rho, -\mathbf{J}, +\phi, -\mathbf{A})_{(-x^0, \mathbf{x}, c)}; \\ T_2 \Phi^e(x^0, \mathbf{x}, c) &= \Phi_{T_2}^e(-x^0, \mathbf{x}, c) = \text{column}(0, -\mathbf{E}, 0, +\mathbf{H}, -\rho, +\mathbf{J}, -\phi, +\mathbf{A})_{(-x^0, \mathbf{x}, c)}; \\ P_1 \Phi^e(x^0, \mathbf{x}, c) &= \Phi_{P_1}^e(x^0, -\mathbf{x}, c) = \text{column}(0, -\mathbf{E}, 0, +\mathbf{H}, +\rho, -\mathbf{J}, +\phi, -\mathbf{A})_{(x^0, -\mathbf{x}, c)}; \\ P_2 \Phi^e(x^0, \mathbf{x}, c) &= \Phi_{P_2}^e(x^0, -\mathbf{x}, c) = \text{column}(0, +\mathbf{E}, 0, -\mathbf{H}, -\rho, +\mathbf{J}, -\phi, +\mathbf{A})_{(x^0, -\mathbf{x}, c)}; \\ Q_1 \Phi^e(x^0, \mathbf{x}, c) &= \Phi_{Q_1}^e(x^0, \mathbf{x}, -c) = \text{column}(0, -\mathbf{E}, 0, -\mathbf{H}, -\rho, -\mathbf{J}, -\phi, -\mathbf{A})_{(x^0, \mathbf{x}, -c)}; \\ Q_2 \Phi^e(x^0, \mathbf{x}, c) &= \Phi_{Q_2}^e(x^0, \mathbf{x}, -c) = \text{column}(0, +\mathbf{E}, 0, +\mathbf{H}, +\rho, +\mathbf{J}, +\phi, +\mathbf{A})_{(x^0, \mathbf{x}, -c)} \end{aligned} \quad (4)$$

The given ratios generalize the ones known in literature [2], [7], [8], [9], [10]. In addition to Maxwell equations they keep invariance of D'Alembert equation for 4-potential and the equation of movement of charged particle in electromagnetic field. By this they form the discrete symmetry

in classical electrodynamics [5]. Due to the ratios

$$\begin{aligned}
P_1^2\Phi &= P_2^2\Phi = T_1^2\Phi = T_2^2\Phi = Q_1^2\Phi = Q_2^2\Phi = E\Phi; \\
P_1P_2\Phi &= T_1T_2\Phi = Q_1Q_2\Phi; \\
[P_1T_1, P_2T_2]\Phi &= [P_1Q_1, P_2Q_2]\Phi = [T_1Q_1, T_2Q_2]\Phi = \\
[P_1T_2, P_2T_1]\Phi &= [P_1Q_2, P_2Q_1]\Phi = [T_1Q_2, T_2Q_1]\Phi = 0
\end{aligned} \tag{5}$$

the number of different symmetries is equal to 16 (coincides with the number of the different combinations of Dirac matrices plus the unit matrix in quantum theory). It is the symmetries

$$\begin{aligned}
E, P_1, P_2, T_1, T_2, Q_1, Q_2, P_1T_1, P_1T_2, P_1Q_1, P_1Q_2, \\
T_1Q_1, T_1Q_2, Q_1Q_2, P_1T_1Q_1, P_1T_1Q_2
\end{aligned} \tag{6}$$

All other combinations, total number of which equals $N = 1 + C_6^1 + C_6^2 + C_6^3 + C_6^4 + C_6^5 + C_6^6 = 64$, can be expressed through combinations (6) for example $P_1Q_1Q_2\Phi = P_2Q_2Q_1\Phi = P_2\Phi$, $P_1P_2T_1T_2\Phi = E\Phi$, $P_1P_2T_1T_2Q_1Q_2\Phi = Q_1Q_2\Phi$ and etc... The symmetries of the type E , P_1 , T_1 , C , CT_1 , CT_2 , P_1T_1 , CP_1T_1 were studied in [1], [2], [3], for example, in connection with physics problems. Additional P_2 and T_2 symmetries were investigated in [7], [8], [10]. Symmetry of the Q type and combinations QT , QP , QPT were discussed in the works [5], [6]. The restriction on the number of symmetries are usually connected with the requirement of scale property of electric charge with respect to space inversion and time reversal, which in symmetry approach is dispensable [7], [8], [10].

Below we shall study the symmetries connected with the inversion of the speed of light.

4 The free Maxwell equations

The inversion of the speed of light is the particular case of discrete transformations of group G_8 and thus forms the symmetry of Maxwell equations. Let us consider the properties of charge conjugation induced by the $c \rightarrow -c$ inversion in case of the free Maxwell equations.

4.1 The charge conjugation in classical sense

We turn attention to the combination Q_1Q_2 which we designate by symbol C_e and which induces the transformations

$$C_e\Phi^e(x^0, \mathbf{x}, c) = \Phi^{-e}(x^0, \mathbf{x}, c) = \text{column}(0, -\mathbf{E}, 0, -\mathbf{H}, -\mathbf{j}, -\rho, -\phi, -\mathbf{A})_{(x^0, \mathbf{x}, c)} \tag{7}$$

Operator C_e change the sign of electrical charge and may be interpreted as the operator of charge conjugation type. In the case of the free fields the charge conjugated function $\Phi_{C_e} = \text{column}(0, -\mathbf{E}, 0, -\mathbf{H}, 0, 0, 0, -\phi, -\mathbf{A})_{(x^0, \mathbf{x}, c)}$ is characterized by the change of the signs of the electric and magnetic fields $\mathbf{E} = \mathbf{l} \exp[-i(k^0x^0 - \mathbf{k}\mathbf{x})]$ and $\mathbf{H} = \mathbf{m} \exp[-i(k^0x^0 - \mathbf{k}\mathbf{x})]$ where $\mathbf{m} = \mathbf{n} \times \mathbf{l}$. By means of replacement $k^0 = \omega/c = \mathcal{E}/\hbar c = p^0/\hbar$, $\mathbf{k} = k^0\mathbf{n} = \mathbf{p}/\hbar$ (ω is the frequency of electromagnetic field, \hbar is the Plank constant, \mathbf{n} is the guiding wave vector, \mathcal{E} is the energy of field, \mathbf{p} is the momentum of field) the result of charge conjugation (7) may be written as follows

$$\begin{aligned}
C_e\Psi_{\mathbf{p} \ \mathbf{n} \ 1}(x^0, \mathbf{x}, c) &= C_e\text{column}(0, \mathbf{E}, 0, \mathbf{H}) = \text{column}(0, -\mathbf{E}, 0, -\mathbf{H}) = \\
&\text{column}(0, -l_1, -l_2, -l_3, 0, -m_1, -m_2, -m_3) e^{-\frac{i}{\hbar}(p^0x^0 - \mathbf{p}\mathbf{x})} = \Psi_{\mathbf{p} \ \mathbf{n} \ -1},
\end{aligned} \tag{8}$$

Here $\Psi \in \Phi$. The charge conjugation C_e changes the signs of polarization vectors \mathbf{l} and \mathbf{m} and keeps the signs of the energy $C_e \mathcal{E} = \mathcal{E}$, momentum $C_e \mathbf{p} = \mathbf{p}$ and guiding wave vector of field $C_e \mathbf{n} = \mathbf{n}$. It is in agreement with the behaviour of the density of the field energy and the field momentum calculated directly from Maxwell equations $W = (E^2 + H^2)/8\pi$, $\mathbf{S} = c(\mathbf{E} \times \mathbf{H})/4\pi$. By this the charge conjugation $C_e = Q_1 Q_2$, being the transformation of symmetry of Maxwell equations, does not result in negative energies. It differs from the charge conjugation C in quantum theory [1], [2]. By given attribute it is possible to identify the C_e conjugation as the charge conjugation in the classical sense. The result of the C_e conjugation coincides with known one [2].

4.2 The charge conjugation in quantum sense

Let us write the Maxwell equations in form of the Dirac equation with help of the 8-dimensional function $\Psi = \text{column}(0, E_1, E_2, E_3, 0, H_1, H_2, H_3)_{(x^0, \mathbf{x}, c)}$ [9]:

$$\gamma^a p_a \Psi(x^0, \mathbf{x}, c) = (i\hbar \gamma^0 \partial_0 + i\hbar \gamma \cdot \nabla) \Psi(x^0, \mathbf{x}, c) = 0 \quad (9)$$

Here $x^a = (ct, x, y, z)$; $g_{ab} = \text{diag}(+, -, -, -)$; $p_a = i\hbar \partial / \partial x^a$; $a, b = 0, k$; $k = 1, 2, 3$; the summation is carried out over the twice repeating index; $\gamma^a = (\gamma^0, \gamma)$; $\gamma \equiv \gamma^k = (\gamma^1, \gamma^2, \gamma^3)$ are the 8-matrices [9]:

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}; \gamma^k = \begin{pmatrix} \alpha^k & 0 \\ 0 & -\alpha^k \end{pmatrix}; \gamma^5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad (10)$$

$$\alpha^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}; \alpha^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}; \alpha^3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad (11)$$

(I -the unit 4-matrix). The gamma matrices have the following properties:

$$\begin{aligned} \gamma^a \gamma^b + \gamma^b \gamma^a &= 2g^{ab}; \quad \gamma^a \gamma^5 + \gamma^5 \gamma^a = -2g^{a0}; \\ (\gamma^0)^+ &= \gamma^0; \quad (\gamma^{1,2,3})^+ = -(\gamma^{1,2,3}); \quad (\gamma^0)^2 = 1; \quad (\gamma^{1,2,3})^2 = -1; \\ (\gamma^{0,1,2,3})^* &= \gamma^{0,1,2,3}; \quad (\gamma^0)^T = \gamma^0; \quad (\gamma^{1,2,3})^T = -\gamma^{1,2,3}; \\ \gamma^5 &= \gamma^0 \gamma^1 \gamma^2 \gamma^3; \quad (\gamma^5)^+ = \gamma^5; \quad (\gamma^5)^* = \gamma^5; \quad (\gamma^5)^2 = 1 \end{aligned} \quad (12)$$

Meaning the conversion of Maxwell equations into Dirac equation, we use the operator of charge conjugation C from quantum theory and define the action of the operator C by analogy with [1]:

$$C \Psi(x^0, \mathbf{x}, c) = U_C \overline{\Psi}^T(x^0, \mathbf{x}, c) = U_C \gamma^0 \Psi^*(x^0, \mathbf{x}, c) \quad (13)$$

Here U_C is the matrix of charge conjugation, $\overline{\Psi} = \Psi^+ \gamma^0$ is the Dirac conjugated function, $*$ is the complex conjugation, T is the transposition.

We take the equation (9) and rewrite it for the Dirac conjugated function $\overline{\Psi}$. Making the transposition, multiplying the result at the left by the matrix U_C and using the property of the gamma-matrices (12) we find:

$$\begin{aligned} \gamma^a p_a \Psi = 0 &\rightarrow \overline{\Psi} \gamma^a p_a = 0 \rightarrow (i\hbar \gamma^{0T} \partial_0 + i\hbar \gamma^T \cdot \nabla) (\Psi^+ \gamma^0)^T \rightarrow \\ &(i\hbar U_C \gamma^{0T} U_C^{-1} \partial_0 + i\hbar U_C \gamma^T U_C^{-1} \cdot \nabla) U_C (\Psi^+ \gamma^0)^T = 0 \end{aligned} \quad (14)$$

It is seen that equation (14) for the charge conjugated function $U_C \bar{\Psi}^T$ coincides with the initial equation (9) if the matrix U_C satisfies the conditions:

$$U_C \gamma^{aT} U_C^{-1} = \gamma^a \rightarrow U_C \gamma^0 U_C^{-1} = \gamma^0; U_C \gamma^k U_C^{-1} = -\gamma^k; k = 1, 2, 3 \quad (15)$$

As far as $U_C \gamma^k + \gamma^k U_C = 0$ in accordance with (12) we can put $U_C = \lambda \gamma^0$, where λ is the factor of proportionality. The expression for the charge conjugated function takes the form:

$$\Psi_C = \lambda \gamma^0 (\Psi^+ \gamma^0)^T = \lambda \gamma^0 \gamma^0 \Psi^* = \lambda \Psi^* = \lambda \text{column}(0, E_1^*, E_2^*, E_3^*, 0, H_1^*, H_2^*, H_3^*) \quad (16)$$

The formula (16) coincides with the result [9] if $\lambda = 1$. Further we write the function describing the initial photon state by analogy with [1], [2]

$$\Psi_{p \mathbf{n} \mathbf{l}} = \frac{1}{\sqrt{2}} \text{column}(0, l_1, l_2, l_3, 0, m_1, m_2, m_3) e^{-\frac{i}{\hbar}(p^0 x^0 - \mathbf{p} \cdot \mathbf{x})}, \quad (17)$$

where $\Psi^+ \Psi = 1$, $p = (\mathcal{E}/c, \mathcal{E} \mathbf{n}/c)$ is the 4-momentum, $\mathcal{E} = \hbar \omega$, $\mathbf{p} = \mathcal{E} \mathbf{n}/c$ are the energy and momentum of photon, \mathbf{n} is the guiding vector of photon, \mathbf{l} is the vector of electrical polarization. Acting on function (17) by the operator C , we find the expression for charge conjugated function in agreement with formula (16):

$$C \Psi_{p \mathbf{n} \mathbf{l}} = \Psi_{-p \mathbf{n} \mathbf{l}} = \frac{\lambda}{\sqrt{2}} \text{column}(0, l_1, l_2, l_3, 0, m_1, m_2, m_3) e^{\frac{i}{\hbar}(p^0 x^0 - \mathbf{p} \cdot \mathbf{x})}, \quad (18)$$

The λ value is equal to $(\pm 1, \pm i)$ as it follows from the condition of the Ψ function normalization. Similarly to the solution of Dirac equation for a particle with nonzero rest mass [1], [2], [3], the photon charge conjugated function is possible to be considered as the function describing a particle with negative energy $\mathcal{E} = -\hbar \omega$ and opposite momentum $\mathbf{p} = -(\hbar \omega/c) \mathbf{n}$. With the field interpretation of the energy and momentum λ is equal $-i$, for example. In this case the field densities of energy and field momentum are $-(E^{*2} + H^{*2})/8\pi$, $-c(E^* X H^*)/4\pi$ i.e. are negative as in quantum theory.

We introduce the designation $\Psi_C = \Psi_{-p \mathbf{n} \mathbf{l}}$ and in spirit of the Dirac interpretation of the solution with negative energy shall consider the Ψ_C solution as the one describing directly not observable vacuum photon. It follows from the formula (18) that besides negative energy and momentum the vacuum photon is characterized by initial sign of guiding vector of propagation \mathbf{n} and by the vectors of electrical and magnetic field polarization \mathbf{l} and \mathbf{m} depending from the choice of the λ value. It follows also from expressions of the quantum 8-currents $j^a = \bar{\Psi} \gamma^a \Psi$ and $j^a_C = \bar{\Psi}_C \gamma^a \Psi$, that the relations $j^0 = j^0_C = 1$, $j^k = j^k_C = (n^1, n^2, n^3)$ take place in agreement with the property of absence of photon charge [2]. The action of the charge conjugation operator C on the charge conjugated function Ψ_C transforms the latter to the function describing the state with positive energy $C \Psi_C = i \gamma^0 (\Psi_C^+ \gamma^0)^T = i \Psi_C^* = \Psi_{p \mathbf{n} \mathbf{l}}$. This state may be interpreted as the antiphoton identical with the photon as result of its neutrality.

Further we shall find how the operator of charge conjugation C may be connected with the operator of conjugation Q induced by the inversion of the speed of light $c \rightarrow -c$.

Let us define the conjugation Q as follows:

$$Q \Psi(x^0, \mathbf{x}, c) = U_Q \bar{\Psi}^T(x^0, \mathbf{x}, -c) = U_Q \gamma^0 \Psi^*(x^0, \mathbf{x}, -c), \quad \hbar \rightarrow -\hbar \quad (19)$$

The inversion of the speed of light and Plank constant do not change the equation (9) because of absence of photon rest mass. Consequently the conjugation Q transforms formally the equation (9) into itself by means of the same matrix as in the case of the charge conjugation $U_Q = U_C = \lambda\gamma^0$. The law of initial photon function $\Psi_{p \text{ n l}}$ transformation is:

$$Q\Psi_{p \text{ n l}} = \Psi_{-p \text{ n l}} = \frac{\lambda}{\sqrt{2}} \text{column}(0, l_1, l_2, l_3, 0, m_1, m_2, m_3) [e^{-\frac{i}{\hbar}(-p^0 x^0 + \mathbf{p}\mathbf{x})}]^* = \frac{1}{\sqrt{2}} \text{column}(0, -l_1, -l_2, -l_3, 0, -m_1, -m_2, -m_3) e^{\frac{i}{\hbar}(p^0 x^0 - \mathbf{p}\mathbf{x} + \hbar\pi/2)} \quad (20)$$

Here we put $\lambda = -i$ and take that with inversion of Plank constant all components of 4-momentum change the signs $i\hbar\partial_a \rightarrow -i\hbar\partial_a$ owing to constancy of the space variables x^a . The obtained result is useful to compare with the result described by formula (8) for the case of charge conjugation in the classical sense. The distinction consists in occurrence in the formula (20) the normalization factor $1/\sqrt{2}$ and sign reversing and displacement of the phase. Besides we should also note that the formula (20) coincides with the formula (18) owing to the following interrelation:

$$C\Psi_{p \text{ n l}}(x^0, \mathbf{x}, c) = Q\Psi_{p \text{ n l}}(x^0, \mathbf{x}, c) \quad (21)$$

The important conclusion may be formed from here. The charge conjugated function describing the vacuum photon state with negative energy on $+c$ hyperplane coincides with the function describing the free photon state with positive energy on the $-c$ hyperplane.

$$\Psi_C(x^0, \mathbf{x}, c) = \Psi_Q(x^0, \mathbf{x}, -c) \rightarrow \Psi_{-p \text{ } -\varepsilon \text{ n l}}(x^0, \mathbf{x}, c) = \Psi_{-p \text{ } +\varepsilon \text{ n l}}(x^0, \mathbf{x}, -c) \quad (22)$$

One can see that the vacuum photon from the $+c$ hyperplane is equivalent to the free photon from the $-c$ hyperplane. It is possible to admit that this is the same object between characteristics of which exist the thin distinctions which depends on its interpretation.

In the case of the vacuum interpretation we may believe that photon is located on $+c$ hyperplane with the positive Plank constant $\hbar > 0$. Its energy is negative $\mathcal{E} = -\hbar\omega < 0$, the frequency is negative $\omega < 0$, the 4-momentum components have the opposite signs $p^a = (-p^0, -\mathbf{p})$. With the field approach the 4-dimensional wave vector components have the opposite signs $k^a = (-\omega/c, -\omega\mathbf{n}/c)$. The photon is in condition of vacuum movement with the positive speed of light and the negative energy.

In the case of the "minus- c " interpretation we may believe that photon is located on the $-c$ hyperplane with the negative Plank constant $\hbar < 0$. Its energy is positive $\mathcal{E} = (-\hbar)(-\omega) > 0$, the frequency is negative $\omega < 0$, the 4-momentum components have the opposite signs $p^a = (-p^0, -\mathbf{p})$. With the field approach the 4-dimensional wave vector components have the initial signs $k^a = (-\omega/(-c), -\omega\mathbf{n}/(-c)) = (\omega/c, \omega\mathbf{n}/c)$. The photon is in condition of the free movement with the negative speed of light and the positive energy.

Both interpretation are in agreement with the ratio (22) which describes the interrelation between the C conjugated state and the Q conjugated state of photon. Their existence are reflected the fact that the charge conjugation in the sense of quantum theory for the case of Maxwell equations can be interpreted also as the consequence of the invariance of these equations with respect to inversion of the speed of light $c \rightarrow -c$. As we can see below, the similar property takes place not only for photon but also for electron states from the Dirac equation [6]. We note only that the operation of the Q conjugation in our case differs from the Q conjugation [6] where

the Plank constant kept the invariant significance. The choice the Q conjugation in form of (13) seems to be more correct for the two reasons.

- In the case of the Plank constant invariance the thin structure constant $\alpha = e^2/\hbar c$ is not invariant as far as $Q(e^2/\hbar c) = -e^2/\hbar c$ that is undesirable.
- In the classical electrodynamics with the non-invariant speed of light the new invariants take place: $\hbar c$ [11] or \hbar/c [12].

Below we select the variant [11] according to which the rest mass of particle m is transformed as $Q(m) = m' = mc^2/(-c)^2 = m$, that is, keeps the invariant significance on the hyperplane $-c$.

5 The Dirac equation

Let us introduce the Dirac equation in the form [1], [2]

$$(\gamma^a p_a - mc)\psi(x^0, \mathbf{x}, c) = (i\hbar\gamma^0\partial_0 + i\hbar\gamma.\nabla - mc)\psi(x^0, \mathbf{x}, c) = 0 \quad (23)$$

Here $x^a = (ct, x, y, z)$; $g_{ab} = \text{diag}(+, -, -, -)$; $\gamma^a = (\gamma^0, \gamma)$; $\gamma = (\gamma^1, \gamma^2, \gamma^3)$ are the Dirac matrices; $p_a = i\hbar\partial/\partial x^a = i\hbar\partial_a$; $a = 0, 1, 2, 3$; the summation is carried out over the twice repeating index; $\Psi = \text{column}(\phi, \chi)$; $\phi = \text{column}(\phi_1, \phi_2)$; $\chi = \text{column}(\chi_1, \chi_2)$. The gamma matrices

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \gamma^{1,2,3} = \begin{pmatrix} 0 & \sigma_{x,y,z} \\ -\sigma_{x,y,z} & 0 \end{pmatrix}; \gamma^5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad (24)$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (25)$$

where I is the unit two dimensional matrix, satisfy the relations [1], [2]:

$$\begin{aligned} \gamma^a\gamma^b + \gamma^b\gamma^a &= 2g^{ab}; \quad \gamma^a\gamma^5 + \gamma^5\gamma^a = 0; \\ (\gamma^0)^+ &= \gamma^0; \quad (\gamma^{1,2,3})^+ = -(\gamma^{1,2,3}); \quad (\gamma^0)^2 = 1; \quad (\gamma^{1,2,3})^2 = -1; \\ (\gamma^{0,1,3})^* &= \gamma^{0,1,3}; \quad (\gamma^2)^* = -\gamma^2; \quad (\gamma^{0,2})^T = \gamma^{0,2}; \quad (\gamma^{1,3})^T = -\gamma^{1,3}; \\ \gamma^5 &= -i\gamma^0\gamma^1\gamma^2\gamma^3; \quad (\gamma^5)^+ = \gamma^5; \quad (\gamma^5)^* = \gamma^5; \quad (\gamma^5)^2 = 1 \end{aligned} \quad (26)$$

In accordance with [1], [2] and formulas (13), (19) we define the action of operators C and Q on the solution ψ in the form:

$$C\psi(x^0, \mathbf{x}, c) = U_C\bar{\psi}^T(x^0, \mathbf{x}, c) = U_C\gamma^0\psi^*(x^0, \mathbf{x}, c); \quad (27)$$

$$Q\psi(x^0, \mathbf{x}, c) = U_Q\bar{\psi}^T(x^0, \mathbf{x}, -c) = U_Q\gamma^0\psi^*(x^0, \mathbf{x}, -c); \quad \hbar \rightarrow -\hbar \quad (28)$$

Here U_C and U_Q are the corresponding matrices of the transformations of the bispinor $\bar{\psi}^T(x^0, \mathbf{x}, c)$ and $\bar{\psi}^T(x^0, \mathbf{x}, -c)$; $\bar{\psi} = \psi^+\gamma^0$; T is the transposition; $*$ is the complex conjugation. We take Eq. (23) and perform the Q -inversion (28) in it:

$$\begin{aligned} Q: (\gamma^a p_a - mc)\psi(x^0, \mathbf{x}, c) = 0 &\rightarrow \bar{\psi}(x^0, \mathbf{x}, -c)(\gamma^a p_a + mc) = 0 \rightarrow \\ (\gamma^{aT} p_a + mc)\bar{\psi}^T(x^0, \mathbf{x}, -c) &\rightarrow (U_Q\gamma^{aT}U_Q^{-1}p_a + mc)U_Q\bar{\psi}^T(x^0, \mathbf{x}, -c) = 0 \end{aligned} \quad (29)$$

Let Eq. (29) coincides with the initial Eq. (23) for the transformed function $\psi_Q = U_Q \bar{\psi}^T$. Matrix U_Q has the following property then:

$$U_Q \gamma^{aT} U_Q^{-1} = -\gamma^a \rightarrow U_Q \gamma^{0,2} = -\gamma^{0,2} U_Q, \quad U_Q \gamma^{1,3} = \gamma^{1,3} U_Q \quad (30)$$

The properties of the U_Q -matrix are the same as the U_C -matrix of the charge conjugation C [1], [2] and we can write:

$$U_Q = U_C = -\gamma^0 \gamma^2 \quad (31)$$

We take into account this relationship and make a table of the transformations of the function $\psi(x^0, \mathbf{x}, c)$ relative to the Q -inversion and the charge conjugation C .

<i>Berestetski, Lifshits, Pitaevski</i>	<i>The present work</i>	
C, P, T symmetries, $c \rightarrow +c$, $\hbar \rightarrow +\hbar$, [2] :	P, T, Q symmetries, $c \rightarrow \pm c$, $\hbar \rightarrow \pm \hbar$:	
$P\psi = i\gamma^0 \psi(x^0, -\mathbf{x}, c);$	$P\psi = i\gamma^0 \psi(x^0, -\mathbf{x}, c);$	
$T\psi = -i\gamma^1 \gamma^3 \psi^*(-x^0, \mathbf{x}, c);$	$T\psi = -i\gamma^1 \gamma^3 \psi^*(-x^0, \mathbf{x}, c);$	
$PT\psi = \gamma^0 \gamma^1 \gamma^3 \psi^*(-x^0, -\mathbf{x}, c);$	$PT\psi = \gamma^0 \gamma^1 \gamma^3 \psi^*(-x^0, -\mathbf{x}, c);$	(32)
$CPT\psi = i\gamma^5 \psi(-x^0, -\mathbf{x}, c);$	$QPT\psi = i\gamma^5 \psi(-x^0, -\mathbf{x}, -c);$	
$CT\psi = i\gamma^1 \gamma^2 \gamma^3 \psi(-x^0, \mathbf{x}, c);$	$QT\psi = i\gamma^1 \gamma^2 \gamma^3 \psi(-x^0, \mathbf{x}, -c);$	
$CP\psi = i\gamma^0 \gamma^2 \psi^*(x^0, -\mathbf{x}, c);$	$QP\psi = i\gamma^0 \gamma^2 \psi^*(x^0, -\mathbf{x}, -c);$	
$C\psi = \gamma^2 \psi^*(x^0, \mathbf{x}, c);$	$Q\psi = \gamma^2 \psi^*(x^0, \mathbf{x}, -c)$	

One can see that the charge conjugation C corresponds to the Q -transformation so that

$$[C, Q]\psi(x^0, \mathbf{x}, c) = 0 \quad (33)$$

Similarly, the operations CPT , CT and CP correspond to the QPT , QT , QP combinations respectively.

Let us consider in more detail the mechanism of the $C \leftrightarrow Q$ correspondence. Following to [2] we rewrite the function ψ in explicit form for our case when $c \neq 1$, $\hbar \neq 1$:

$$\psi_{p\sigma} = \frac{1}{\sqrt{2p^0}} u_{p\sigma} e^{-\frac{i}{\hbar} p \cdot x}; \quad \psi_{-p-\sigma} = \frac{1}{\sqrt{2p^0}} u_{-p-\sigma} e^{\frac{i}{\hbar} p \cdot x}; \quad (34)$$

$$u_{p\sigma} = \begin{pmatrix} \frac{\sqrt{p^0 + mc}}{\sqrt{p^0 - mc}} w \\ (\mathbf{n}\sigma)w \end{pmatrix}; \quad u_{-p-\sigma} = \begin{pmatrix} \frac{\sqrt{p^0 - mc}}{\sqrt{p^0 + mc}} w' \\ (\mathbf{n}\sigma)w' \end{pmatrix} \quad (35)$$

Here $p^0 = \mathcal{E}/c > 0$, $\mathcal{E} = c\sqrt{\mathbf{p}^2 + m^2 c^2} > 0$, $p \cdot x = p^0 x^0 - \mathbf{p} \cdot \mathbf{x}$, $\mathbf{n} = \mathbf{p}/p$, $w^+ w = 1$, $w = (\mathbf{n}\sigma)w'$, $\bar{u}_p u_p = 2mc$, $\bar{u}_{-p} u_{-p} = -2mc$, $c > 0$. The result of operators C and Q application to the function $\psi_{p\sigma}$ is:

$$\begin{aligned} C\psi_{p\sigma\mathcal{E}}(x^0, \mathbf{x}, c) &= \gamma^2 \psi_{p\sigma\mathcal{E}}^*(x^0, \mathbf{x}, c)_{(c \rightarrow c \quad \hbar \rightarrow \hbar \quad \sigma \rightarrow \sigma)} = \\ &= \frac{1}{\sqrt{2p^0}} \begin{pmatrix} 0 & \sigma_y \\ -\sigma_y & 0 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{p^0 + mc}}{\sqrt{p^0 - mc}} w^* \\ (\mathbf{n}\sigma^*)w^* \end{pmatrix} (e^{-\frac{i}{\hbar}(p^0 x^0 - \mathbf{p} \cdot \mathbf{x})})^* = \\ &= \frac{1}{\sqrt{2p^0}} \begin{pmatrix} \frac{\sqrt{p^0 - mc}}{\sqrt{p^0 + mc}} (\mathbf{n}\sigma)w' \\ (\mathbf{n}\sigma)w' \end{pmatrix} e^{\frac{i}{\hbar}(p^0 x^0 - \mathbf{p} \cdot \mathbf{x})} = \frac{u_{-p-\sigma}}{\sqrt{2p^0}} e^{\frac{i}{\hbar}(p^0 x^0 - \mathbf{p} \cdot \mathbf{x})} = \psi_{-p-\sigma-\mathcal{E}}(x^0, \mathbf{x}, c) \end{aligned} \quad (36)$$

$$\begin{aligned}
Q\psi_{p\sigma\mathcal{E}}(x^0, \mathbf{x}, c) &= \gamma^2 \psi_{p\sigma\mathcal{E}}^*(x^0, \mathbf{x}, c)_{(c \rightarrow -c \quad \hbar \rightarrow -\hbar \quad \sigma \rightarrow -\sigma)} = \\
&= \frac{1}{i\sqrt{2p^0}} \begin{pmatrix} 0 & -\sigma_y \\ \sigma_y & 0 \end{pmatrix} \begin{pmatrix} -i\sqrt{p^0 + mc} w^* \\ -i\sqrt{p^0 - mc} (\mathbf{n}\sigma^*) w^* \end{pmatrix} (e^{-\frac{i}{\hbar}(-p^0 x^0 + \mathbf{p}\mathbf{x})})^* = \\
&= \frac{1}{\sqrt{2p^0}} \begin{pmatrix} \sqrt{p^0 - mc} (\mathbf{n}\sigma) w' \\ \sqrt{p^0 + mc} w' \end{pmatrix} e^{\frac{i}{\hbar}(p^0 x^0 - \mathbf{p}\mathbf{x})} = \frac{u_{-p-\sigma}}{\sqrt{2p^0}} e^{\frac{i}{\hbar}(p^0 x^0 - \mathbf{p}\mathbf{x})} = \psi_{-p-\sigma+\mathcal{E}}(x^0, \mathbf{x}, -c)
\end{aligned} \tag{37}$$

We take into account in the first expression that $\sigma_y(\mathbf{n}\sigma^*) = -(\mathbf{n}\sigma)\sigma_y$, $-\sigma_y w^* = w'$, $w'^+ w' = (\sigma_y w^*)^+ (\sigma_y w^*) = w^T \sigma_y^+ \sigma_y w^* = (w^+ w)^* = 1$ and in addition to this in the second expression that $p^0 = \mathcal{E}/(-c) < 0$, $\mathbf{p} = \mathcal{E}(-\mathbf{v})/c^2 < 0$. Analogously to result (21) for photon states we have for electron ones

$$C\psi_{p\sigma\mathcal{E}}(x^0, \mathbf{x}, c) = Q\psi_{p\sigma\mathcal{E}}(x^0, \mathbf{x}, c) \rightarrow \psi_{-p-\sigma-\mathcal{E}}(x^0, \mathbf{x}, c) = \psi_{-p-\sigma+\mathcal{E}}(x^0, \mathbf{x}, -c) \tag{38}$$

The difference consists in that the electromagnetic field describes the neutral particles (photons) and the $\psi_{p\sigma\mathcal{E}}$ field describes the charge particles (electrons, positrons). We consider this case in more detail.

5.1 The Dirac equation for a charged particle

Let us introduce the Dirac equation for a charge particle with spin 1/2 in electromagnetic field:

$$(\gamma^a p_a - mc)\psi(x, c) = (e/c)\gamma^a A_a \psi(x, c) \tag{39}$$

where $x = (x^0, \mathbf{x})$, e is the charge of a particle, $A^a = (A^0, \mathbf{A})$ is the 4-potential, γ -are the matrices (24). Let us subject the equation (39) to the Q transformation taking into account the formulas (28), (31) on the assumption that an electrical charge is a scalar; the vector - potential is a polar vector relative to the replacements both $\mathbf{x} \rightarrow -\mathbf{x}$, and $t \rightarrow -t$, and $c \rightarrow -c$ [6].

$$\begin{aligned}
Q(x^0, \mathbf{x}, c, e, A^0, \mathbf{A}) &= (x^0, \mathbf{x}, -c, e, A^0, \mathbf{A}); \\
Q\psi(x^0, \mathbf{x}, c) &= \psi_Q(x^0, \mathbf{x}, -c) = U_Q \bar{\Psi}^T(x^0, \mathbf{x}, -c)
\end{aligned} \tag{40}$$

Taking the Dirac conjugate equation, making the transposition, multiplying on the matrix U_Q at the left and using the properties of the γ -matrices (26) we have

$$\begin{aligned}
Q : (\gamma^a p_a - mc)\psi &= (e/c)\gamma^a A_a \psi \rightarrow \\
\bar{\psi}(x^0, \mathbf{x}, -c)(\gamma^a p_a + mc) &= -(e/c)\bar{\psi}(x^0, \mathbf{x}, -c)(A^0 \gamma^0 - \mathbf{A} \cdot \boldsymbol{\gamma}) \rightarrow \\
(i\hbar \gamma^{0T} + i\hbar \boldsymbol{\gamma}^T \cdot \nabla + mc)\bar{\psi}^T(x^0, \mathbf{x}, -c) &= -(e/c)(\gamma^{0T} A^0 - \boldsymbol{\gamma}^T \mathbf{A}) \rightarrow \\
(i\hbar U_Q \gamma^{0T} U_Q^{-1} + i\hbar U_Q \boldsymbol{\gamma}^T U_Q^{-1} \cdot \nabla + mc)U_Q \bar{\psi}^T(x^0, \mathbf{x}, -c) &= \\
-(e/c)(U_Q \gamma^{0T} U_Q^{-1} A^0 - U_Q \boldsymbol{\gamma}^T U_Q^{-1} \mathbf{A})U_Q \bar{\psi}^T(x^0, \mathbf{x}, -c) &\rightarrow \\
(\gamma^a p_a - mc)\psi_Q(x^0, \mathbf{x}, -c) &= -(e/c)\gamma^a A_a \psi_Q(x^0, \mathbf{x}, -c)
\end{aligned} \tag{41}$$

Here matrix U_Q satisfies the conditions (30) which define its explicit form (31). Taking into account $\psi_Q = \gamma^2 \psi^*(x^0, \mathbf{x}, -c)$ we can write

$$(\gamma^a p_a - mc)\gamma^2 \psi^*(x^0, \mathbf{x}, -c) = (-e/c)\gamma^a A_a \gamma^2 \psi^*(x^0, \mathbf{x}, -c) \tag{42}$$

Similarly to the charge conjugation, the equation received coincides with initial Eq. (39) for the electric charge $-e$ and the transformed function $\gamma^2\psi^*(x^0, \mathbf{x}, -c)$. In accordance with formulae (36), (37) and (38) it is possible to admit that Eq. (42) describes a particle with the charge $-e$, 4-momentum $p = (-p^0, -\mathbf{p})$ and positive energy $+\mathcal{E}$ ($-p^0 = (+\mathcal{E})/(-c)$, $-\mathbf{p} = (+\mathcal{E})(-\mathbf{v})/c^2$).

Thus, the charge conjugation C puts into correspondence the antiparticle with characteristics $(-e, -m, -p, -\mathcal{E}, c)$ from hyperplane $+c$ to the particle with characteristics $(e, m, p, \mathcal{E}, c)$. The Q -conjugation puts into correspondence the particle with characteristics $(-e, m, -p, \mathcal{E}, -c)$ from hyperplane $-c$ to the particle with characteristics $(e, m, p, \mathcal{E}, c)$. It is seen that the particle from hyperplane $-c$ with characteristics $(-e, m, -p, \mathcal{E}, -c)$ may be the redefined antiparticle with respect to the initial particle from Eq. (39). It is the same particle which we interpret as the antiparticle on hyperplane $+c$.

As in the case of the C -conjugation [2], the Q -conjugation forms the symmetry transformation of Dirac equation (39) for a charged particle if the 4-potential of electromagnetic field A is transformed to the rule $Q(A) = (-A^0, -\mathbf{A})$.

6 Conclusion

The inversion of the speed of light $x^0 \rightarrow x^0, \mathbf{x} \rightarrow \mathbf{x}, c \rightarrow -c$ was considered in the Maxwell and the Dirac equations. It is shown that the charge conjugation C_e in classical sense and the charge conjugation C in quantum sense can be interpreted as the consequence of the symmetry of these equations with respect to the discrete transformation $c \rightarrow -c$.

Among consequences of the $c \rightarrow -c$ symmetry we can note the following ones.

In accordance with classical and quantum electrodynamics we can admit that the world as a whole is the five-dimensional one. It consists of two hyperplanes: $(x^0, \mathbf{x}, +c)$ and $(x^0, \mathbf{x}, -c)$, where $c = 3 \cdot 10^{10}$ cm/s. Each of the hyperplanes forms the 4-dimensional Minkowski subspace with metric tensor $g_{ab} = \text{diag}(+, -, -, -)$ and each of the hyperplanes is filled with photons and electrons.

The equations of classical and quantum electrodynamics are the same for $+c$ and $-c$ hyperplanes. The electron-photon Dirac vacuum exists on the each hyperplane. The free photon with positive energy from $-c$ hyperplane is the same object as the vacuum photon with negative energy from $+c$ hyperplane. The free electron with positive energy from hyperplane $-c$ is the same object as the vacuum electron with negative energy from $+c$ hyperplane. The $-c$ hyperplane is equivalent to the Dirac vacuum from $+c$ hyperplane, or the $-c$ hyperplane show himself as the Dirac vacuum. Therefore further we can deal with the Dirac vacuum as with the more known object.

With this point of view the reaction $e^- + e^+ \rightarrow \gamma + \gamma$ can be interpreted as the act of annihilation of the electron e^- and the electron hole e^+ [1]. Analogously, the reaction $\gamma + \gamma \rightarrow e^- + e^+$ we can interpret as the act of annihilation of the uncharged photon and uncharged photon hole. In the case of the electron vacuum the vacuum transitions $e^- \rightarrow e'^- + \gamma$ are impossible because of the Pauli principle [1]. In the case of the photon vacuum the vacuum transitions $\gamma \rightarrow \gamma' + e^- + e^+$ are impossible due to the law of energy-momentum conservation, because of the equations of the energy-momentum conservation $-\hbar\omega = -\hbar\omega' + \mathcal{E}^{(e)} + \mathcal{E}^{(p)}$; $-\hbar\omega\mathbf{n}/c = -\hbar\omega'\mathbf{n}'/c + \mathbf{p}^{(e)} + \mathbf{p}^{(p)}$ are non-consistent in general. Here $-\hbar\omega$ and $-\hbar\omega'$ are the negative energies of the vacuum photons with the frequencies $\omega' > \omega > 0$; $\mathcal{E}^{(e)} > 0$ and $\mathcal{E}^{(p)} > 0$ are the positive energies of the free

electron and positron; \mathbf{n} and \mathbf{n}' are the guiding vectors of the vacuum photons; $\mathbf{p}^{(e)}$ and $\mathbf{p}^{(p)}$ are the momenta of the free electron and positron. As this takes place, we are going from hypothesis that the transitions of vacuum electrons are accompanying by generation of the free photons and the transitions of vacuum photons are accompanying by generation of the free electron-positron pairs or another pairs particle-antiparticle with total zero charge.

In the same time spontaneous photon transitions from the states with positive energy to the states with negative energy are possible as the reaction of annihilation $\gamma + \gamma \rightarrow \nu + \bar{\nu}; e^- + e^+; \dots$. Products of annihilation (neutrino and antineutrino, electron and positron and etc.) may annihilate in turn and generate the new gamma quanta. As a result of this the continuous process of energy exchange will be established between the vacuum and real world, or in other words, between the $-c$ hyperplane and the $+c$ hyperplane. The electron-photon vacuum acquires dynamical character. Perhaps this is just the physical sense of the "minus c " symmetry in addition to the interrelation between C and Q conjugations.

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